**SPRING MID SEMETER EXAMINATION-2022**

**B.Tech   
4th Semester (*Regular) SAS-2023(SET-01)***



**Subject: Discrete Mathematical Structure**

**Code: MA2013**

**Full Marks: 20 Time: 1.5 Hrs**

**Answer any four questions including question No. 1 which is compulsory.** **The figures in the margin indicate full marks.**

**Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only**

|  |  |  |  |
| --- | --- | --- | --- |
| 1. | Answer the following questions | | [5×1=5] |
|  | (a) | What is negation of the statement “All poor people are sad.”  Ans: Domain be all poor people. P(x): x is sad, given statement is  Negation is  That is, Some poor people are not sad. |  |
|  | (b) | State which rule of inference is used in the argument: “If it rains today, the University will close. The University is not closed today. Therefore, it did not rain today.”  Ans: Let p: It rains today, q: The University will close  Given argument is  This argument is valid and the rule is Modus Tollens. |  |
|  | (c) | Translate the English sentence into propositional logic. “Fido is neither a dog nor a cat, but rather a goose.”  Ans: Let p: Fido is neither a dog, q: Fido is a cat, r: Fido is a goose  The given statement is . |  |
|  | (d) | Write the no of reflexive and symmetric relations of a set with distinct elements.  Ans: |  |
|  | (e) | Express the proof technique in mathematical induction as a rule of inference.  Ans:  The rule of inference is |  |
|  |  |  |  |
| 2. | (a) | Suppose is a real number. Consider the statement “If , then .” Construct its converse, inverse, and contrapositive. Also determine the truth values of each of the statement.  Ans: Let p: , q:  Given statement is  Truth value of the statement is F.  Converse is  , “If , then .”  Truth value of converse is T.  Inverse is  , “If , then .”  Truth value of inverse is T.  Contrapositive is  , “If , then .”  Truth value of contrapositive is F. | [3] |
|  | (b) | Show that the statements and are logically equivalent.  Ans: Construct the truth table:   |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | p | q |  |  |  |  |  |  | | T | T | F | F | T | F | F | F | | T | F | F | F | T | F | T | F | | F | T | T | T | T | F | F | F | | F | F | T | F | F | T | T | T |   From the truth table it is observed that both values in columns andare same. So, they arelogically equivalent | [2] |
|  |  |  |  |
| 3. | (a) | What is strong mathematical induction? Use it to prove for all , where and for .  Ans: strong mathematical induction is the rule  Let p(n):  Given  for .  where .  Now . So p(1) is true.  Assume p(2), p(3)…p(k) are true.  To show p(k+1) is true  As p(k-1) and p(k) are true, and  Now  Thus  Hence P(k+1) is true.  Therefore, by strong mathematical induction is true.  That is, for all | [3] |
|  | (b) | Use De Morgan’s law to write the negation the following statement, simplifying so that only simple statements are negated. “If Phoebe buys a pizza, then Calvin buys popcorn.”  Ans: let p: Phoebe buys a pizza and q: Calvin buys popcorn  Given statement is  So its negation is  That is, Phoebe buys a pizza but, Calvin does not buy popcorn. | [2] |
|  |  |  |  |
| 4. | (a) | Given is a relation on the set of integers defined by . Is the relation reflexive, symmetric, antisymmetric, or transitive?  Ans: For any integer is divisible by 4, that is so and hence R is reflexive.  Let , then implies is divisible by 4. Then is divisible by 4, that is . Thus symmetric,  Now and but , So R is not antisymmetric.  Let . Then and . That is is divisible by 4 and is divisible by 4. Therefore is divisible by 4. That is . So . Hence R is Transitive. | [3] |
|  | (b) | How many integers are there between and which are not divisible by .  Ans:  Let be the set of integers between and divisible by .  Then  By set inclusion and exclusion we have    So that are not divisible by is | [2] |
|  |  |  |  |
| 5. | (a) | Show that the premises “A student in this class has not read the book” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not read the book”.  Ans: Let domain be all students of this class.  P(x) : x has read the book.  Q(x): x passed the first exam.  Given argument is of the form  As  for some c in this class by Existential instantiation.  Now  By Universal instantiation. Thus  By conjunction. Finally,  By Existential generalization*.* | [3] |
|  | (b) | If and are relations defined on the set of integers given by R and then find , , and .  Ans: | [2] |

\*\*\*\*\*